Milanković Cycles and the Earth-Jupiter-Sun Interaction

Introduction

The Earth, like the other planets, moves in an elliptical orbit. We are closest to the Sun in January, and furthest away in July. If The Earth were the only planet in the solar system, this orbit would be perfectly stable. However, the influence of other planets causes the shape of the orbit to change slowly, over a period of thousands of years. The same is of course true for the other planets.

It is impossible to find an analytical solution for the motion of 3 gravitating bodies, and trying to describe precisely the motion of the solar system was one of the main reason Newton and others following him developed the numerical methods we are using in this course. Of course, they did not have access to computers, and had to make do with vast quantities of paper and ink!

The changes in the Earth's orbit due to the Moon and the other planets have an importance beyond just getting precise astronomical data. The Serbian engineer Milutin Milanković proposed that the ice age cycles can to a large extent be explained by these changes. He identified a number of cycles, with periods of 40,000 up to 400,000 years, in the Earth's orbit and rotation, and linked these to global temperature variations.

In this project we will study the variations in the eccentricity of the Earth's orbit, which can to a large extent be explained by the influence of the largest planet (and the closest of the gas giants) in the solar system, Jupiter.

The 2-body problem

Since the Milanković cycles occur over a very long time span, it is important to make sure that our ODE solver is stable over this time span. We will make some changes to the Earth's actual orbit and the mass of Jupiter to allow us to see changes over shorter time spans, but we will still want our results to be reliable over a period of 10,000 years. Your first task will therefore be to study the motion of the Earth around the Sun on its own, and if necessary adjust the tolerances or step sizes in your ODE solver to ensure the Earth maintains a stable elliptical orbit for at least 10,000 years.

The 2-body central force problem is usually written in polar coordinates, but the 3-body problem is much more easily formulated in cartesian coordinates, so you should use the cartesian equations of motion:

$$\ddot{x} = -GM_\odot \frac{x}{(x^2 + y^2)^{3/2}}, \tag{1}$$

$$\ddot{y} = -GM_\odot \frac{y}{(x^2 + y^2)^{3/2}}. \tag{2}$$

Choosing appropriate units is an important part of numerical problem-solving, since we do not want our numbers to be too small or too big. In this case, it will be convenient to measure time in years (yr), distance in astronomical units (1AU = the mean distance, or semimajor axis, between the Earth and the Sun), and mass in solar masses ($M_\odot$). In these units, Newton's gravitational constant is $G = 58.9639\text{AU}^2/M_\odot\text{yr}^2$.

Your first task is to reconstruct the Earth's orbit. You can take the semimajor axis of the orbit to be along the x-axis, and start at $t = 0$ with the Earth at its aphelion $x = 1.016714$AU. You should then find the inital velocity v_y which will give an elliptic orbit with semimajor axis 1.000000AU and eccentricity 0.01671123.

Using `odeint` or your own Runge–Kutta integrator, integrate (1), (2) with these inital conditions. You should get a stable orbit with a period of 1 year. Integrate the orbit for 10,000 years, and adjust the relative and absolute tolerances to ensure the orbit remains stable for this length of time.

You can also vary the initial velocity of the Earth to make the orbit more eccentric. This will make it easier to see the changes due to the influence of Jupiter in the next section.

The 3-body problem

You will now be ready to study the interaction of the Sun, the Earth and Jupiter. We will take the Sun to be fixed at the origin of our coordinate system: since it is far heavier than even Jupiter, it will not be affected much by the motion of the planets.

First of all, you can use the equations (1), (2) to study the orbit of Jupiter on its own, by replacing the orbit parameters of the Earth with those of Jupiter. You can start with Jupiter at its aphelion, $y = 4.950429$AU. The semimajor axis is 5.204267AU, and the eccentricity is 0.048775. The orbital period is 11.859 years. You should find that the orbit of Jupiter will be stable if you use the same tolerances or step size as for the Earth's orbit.

Now, write a set of coupled differential equations for the motion of the Earth and Jupiter. This will be a set of 8 coupled first-order equations, derived from the 4 second-order equations (which you should obtain yourselves to check you understand them):

$$\ddot{x_\oplus} = -GM_\odot \frac{x_\oplus}{(x_\oplus^2 + y_\oplus^2)^{3/2}} - GM_J \frac{x_\oplus - x_J}{\left((x_\oplus - x_j)^2 + (y_\oplus - y_J)^2\right)^{3/2}}, \tag{3}$$

$$\ddot{y_\oplus} = -GM_\odot \frac{y_\oplus}{(x_\oplus^2 + y_\oplus^2)^{3/2}} - GM_J \frac{y_\oplus - y_J}{\left((x_\oplus - x_J)^2 + (y_\oplus - y_J)^2\right)^{3/2}}, \tag{4}$$

$$\ddot{x_J} = -GM_\odot \frac{x_J}{(x_J^2 + y_J^2)^{3/2}} + GM_\oplus \frac{x_\oplus - x_J}{\left((x_\oplus - x_J)^2 + (y_\oplus - y_J)^2\right)^{3/2}}, \tag{5}$$

$$\ddot{y_J} = -GM_\odot \frac{y_J}{(x_J^2 + y_J^2)^{3/2}} + GM_\oplus \frac{x_\oplus - x_J}{\left((x_\oplus - x_J)^2 + (y_\oplus - y_J)^2\right)^{3/2}}, \tag{6}$$

$$\tag{7}$$

The mass of the Earth $M_\oplus$ and the mass of Jupiter M_J are

$$M_\oplus = \frac{1}{332950} M_\odot \,, \qquad M_J = \frac{1}{1048} M_\odot \,. \tag{8}$$

If, to start with, you make Jupiter 10 times more massive than it actually is, and the Earth's orbit somewhat more eccentric than it is in reality, you should see changes in the Earth's orbit over a period of a few hundred to a few thousand years. The orbit of Jupiter should be unaffected. Try to quantify these changes:

- How much does the eccentricity change?
- How much does the perihelion position change?
- Does the semimajor axis change?

You can for example use `odeint`'s event finder function to determine the perihelion and aphelion distance and position, and thereby the eccentricity and semimajor axis.

If you are now feeling brave, you can try to make the parameters more realistic. You may have to go off and have dinner or go home to sleep while waiting for the run to finish, if you want to do a 100,000 year run.

Discuss the changes you see in the orbits, and the significance of these changes.

References

[1] http://en.wikipedia.org/wiki/Milankovich_cycles